M. Sc.

Sem I

3rd paper Linear Algebra

Skew Hermitian matrix (SHM)

A square matrix $A = [a_{ij}]$ is called SHM if

$$\boxed{a_{ij} = -\overline{a_{ji}}} \quad \forall i, \forall j$$

i.e. the (i,j)th element of A is equal to the negative of the conjugate complex of the (j,i)th element.

$\because \quad a_{ij} = -\overline{a_{ji}}$

$\Rightarrow \quad a_{ij} + \overline{a_{ji}} = 0.$

Again, replacing j by i,

$$a_{ii} + \bar{a}_{ii} = 0$$

$\Rightarrow a_{ii} (\forall i)$ must be either a ~~const~~ pure imaginary number or zero.

$\Rightarrow$ All the diagonal elements ($a_{11}, a_{22}, a_{33} \ldots$) of a skew Hermitian matrix must be purely imaginary or zero.

Examples

$$A = \begin{bmatrix} 0 & -5-2i \\ 5-2i & 0 \end{bmatrix}$$

Here, $a_{11} = 0 = a_{22}$, $a_{12} = -5-2i$

$\therefore \bar{a}_{12} =$ conjugate of $-5-2i = -5+2i$

$\therefore (5-2i) + (-5+2i) = 0$.

i.e. $a_{21} + \bar{a}_{12} = 0$

Also, $a_{12} + \bar{a}_{21} = (-5-2i) + (5+2i) = 0$

Hence, A is skew symmetric.